Name: Solutions   Section: ______

Proofs about Subsets

Prove if $x \in \overline{A \cup B}$, then $x \in \overline{A} \cup \overline{B}$

1. Let A, B be arbitrary sets. Prove that $\overline{A \cup B} \subseteq \overline{A} \cup \overline{B}$

Suppose $x \in \overline{A \cup B}$

then $\neg(x \in A \cup B)$

so $\neg(x \in A$ <u>or</u> $x \in B)$

so $\neg(x \in A)$ <u>and</u> $\neg(x \in B)$

so $x \in \overline{A}$ <u>and</u> $x \in \overline{B}$

so $x \in \overline{A}$

so $x \in \overline{A}$ <u>or</u> $x \in \overline{B}$

Therefore, $x \in \overline{A} \cup \overline{B}$ ☺

we have proved if $x \in \overline{A \cup B}$ then $x \in \overline{A} \cup \overline{B}$

Therefore, $\overline{A \cup B} \subseteq \overline{A} \cup \overline{B}$.

Prove if $x \in A - C$ then $x \in A \cup B$

2. Let A, B, C be arbitrary sets. Prove that $A - C \subseteq A \cup B$

<u>Suppose</u> $x \in A - C$

so $x \in A$ <u>and</u> $x \notin C$

so $x \in A$

so $x \in A$ <u>or</u> $x \in B$

Therefore, $x \in A \cup B$ ☺

Prove if $x \in (A \cup B) - C$ then $x \in A \cup (B - C)$

3. Let A, B, C be arbitrary sets. Prove that $(A \cup B) - C \subseteq A \cup (B - C)$

<u>Suppose</u> $x \in (A \cup B) - C$

so $(x \in A$ <u>or</u> $x \in B)$ <u>and</u> $x \notin C$

There are two cases:

<u>Case 1</u>: $x \in A$	<u>Case 2</u>: $x \in B$
then $x \in A$ <u>or</u> $x \in B - C$	<u>Recall</u> $x \notin C$
so $x \in A \cup (B - C)$	so $x \in B$ and $x \notin C$
	so $x \in B - C$
	so $x \in A$ or $x \in B - C$

in either case, $x \in A \cup (B - C)$ ☺

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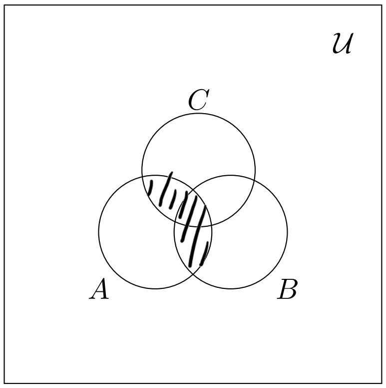
Combining Sets

1. Let A, B, C be sets. Shade the Venn Diagram for $A \cap (B \cup C)$.

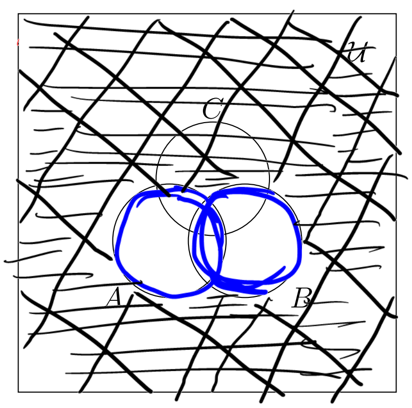
   Based on your drawing, do you think that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$?



$x \in A \cap (B \cup C) \iff x \in A$ <u>and</u> $x \in B \cup C$

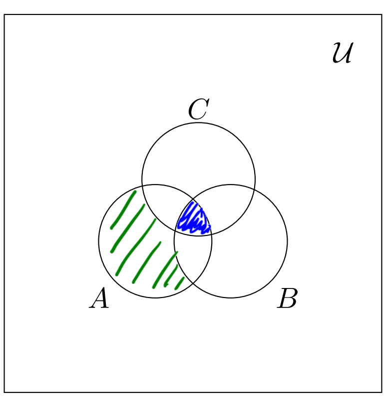
2. Let A, B, C be sets. Shade the Venn Diagram for $\overline{A \cup B}$.

   Based on your drawing, do you think that $\overline{A \cup B} = \overline{A} \cap \overline{B}$?



$x \in \overline{A \cup B} \iff \neg(x \in A \text{ or } x \in B)$

3. Let A, B, C be sets. Shade the Venn Diagram for $(A - (B \cup C)) \cup (A \cap B \cap C)$.



$\underbrace{(x \in A \wedge x \notin B \cup C)}$ or $\underbrace{(x \in A \cap B \cap C)}$

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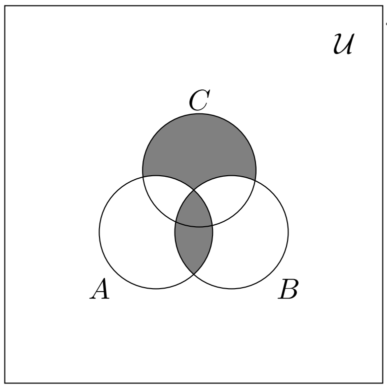
4. Describe the shaded region two ways: First, describe its elements in words. Then write the set in terms of A, B, C using set operations.



elements:

$x \in C$ and $x \notin A \cup B$

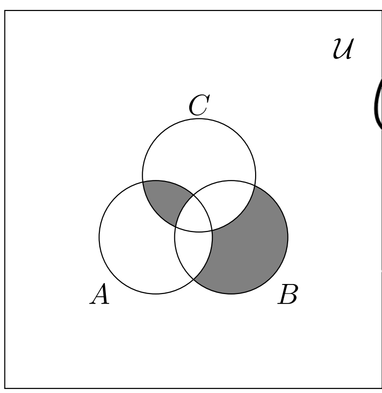
or

$x \in A$ and $x \in B$

as a set:

$(C - (A \cup B)) \cup (A \cap B)$

5. Describe the shaded region two ways: First, describe its elements in words. Then write the set in terms of A, B, C using set operations.



elements

$(x \in A$ and $x \in C)$ and $x \notin B$

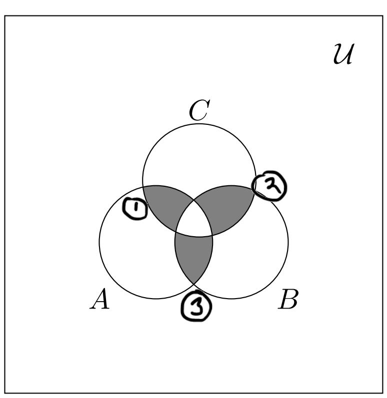
or

$x \in B$ and $x \notin A \cup C$

as a set

$((A \cap C) - B) \cup (B - (A \cup C))$

6. Describe the shaded region two ways: First, describe its elements in words. Then write the set in terms of A, B, C using set operations.



elements: $x \in A \cap C$ and $x \notin B$

or $x \in B \cap C$ and $x \notin A$

or $x \in A \cap B$ and $x \notin C$

as a set:

$((A \cap C) - B) \cup ((B \cap C) - A) \cup ((A \cap B) - C)$

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7. A survey of 80 people indicated the following 26 read Jack Kerouac, 41 read Shakespeare, 18 read Chinua Achebe, 7 read Shakespeare and Kerouac, 12 read Achebe and Kerouac, 9 read Shakespeare and Achebe, and 4 read all three authors.

   Questions:

   (a) Draw and fill in a Venn diagram to illustrate the given information.

   (b) How many people in the survey read none of the three authors?

   (c) How many read either Shakespeare or Achebe?

   (d) How many read Shakespeare but not Kerouac?

   (e) How many read Achebe and Kerouac but not Shakespeare?

8. A survey of 75 people indicated the following 14 read Natsume Soseki, 39 read Maya Angelou, 23 read Augustine of Hippo, 5 read Soseki and Angelou, 17 read Angelou and Augustine, 8 read Soseki and Augustine, and 3 read all three authors.

   Questions:

   (a) Draw and fill in a Venn diagram to illustrate the given information.

   (b) How many people in the survey read none of the three authors?

   (c) How many read either Angelou or Augustine?

   (d) How many read Soseki but not Augustine?

   (e) How many read Soseki and Augustine but not Angelou?

9. A survey of 100 people indicated the following 53 liked chicken, 34 liked sushi, 22 liked tofu, 27 liked chicken and sushi, 9 liked sushi and tofu, 16 liked chicken and tofu, and 5 liked all three foods.

   Questions:

   (a) (a) Draw and fill in a Venn diagram to illustrate the given information.

   (b) How many people in the survey liked none of the three choices?

   (c) How many liked chicken or tofu?

   (d) How many liked sushi but not tofu?

   (e) How many liked sushi and tofu but not chicken?

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10. Write down a formula for $|A \cup B \cup C|$ in terms of $|A|, |B|, |C|, |A \cap B|, |A \cap C|, |B \cap C|, |A \cap B \cap C|$.

    Hint: draw a Venn Diagram. Start by determining how many elements will be double or triple-counted by $|A| + |B| + |C|$, and then adjust your formula as you go. You will need to adjust it more than once!

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It is nice to have a description of the whole universe $\mathcal{U}$ that does *not* double count any element. This leads us to the following definition.

Define: We say that a collection $S_1, \ldots, S_n$ of subsets of $\mathcal{U}$ is a *partition* of $\mathcal{U}$ if

(a) The sets have no elements in common. In other words, if $S_i \cap S_j = \emptyset$ for all $i \neq j$, and

(b) The union of the sets is everything. In other words, if $\mathcal{U} = S_1 \cup S_2 \cup \cdots \cup S_n$

For example, the Venn Diagram of A, B, C cuts up the universe $\mathcal{U}$ into a collection of 8 separate subsets that are disjoint (share no elements) and that union up to everything.

(Draw the picture and label the components of the partition!)

11. Let $A = \{a, b, c, d, e, f, g\}$.

Which of the following are partitions of A?

(a) $\underbrace{\{a, c, g\}}_{S_1}, \underbrace{\{b, e\}}_{S_2}, \underbrace{\{d\}}_{S_3}$

Not a partition. the sets are disjoint
But $f \notin S_1 \cup S_2 \cup S_3$

(b) $\underbrace{\{a, c, f, g\}}_{S_1}, \underbrace{\{b, c, e\}}_{S_2}, \underbrace{\{d\}}_{S_3}$

Not a partition. $S_1 \cap S_2 = \{c\}$ is NOT empty.

(c) $\underbrace{\{a, c, f, g\}}_{S_1}, \underbrace{\{b, e\}}_{S_2}, \underbrace{\{d\}}_{S_3}$

IS a partition. NO element is in two S_i
AND
every element is in some S_i

(d) $\underbrace{\{a, b\}}_{S_1}, \underbrace{\{c, f\}}_{S_2}, \underbrace{\{g\}}_{S_3}, \underbrace{\{d, e\}}_{S_4}$

IS a partition. NO element is in two S_i
AND
every elt is in some S_i

(e) $\underbrace{\{a, b, c, d, e, f, g\}}_{S_1}$

IS a partition. NO elt is in two S_i
AND
every elt is in some S_i

(f) $\underbrace{\{a\}}_{S_1}, \underbrace{\{b\}}_{S_2}, \underbrace{\{c\}}_{S_3}, \underbrace{\{d\}}_{S_4}, \underbrace{\{e\}}_{S_5}, \underbrace{\{f\}}_{S_6}, \underbrace{\{g\}}_{S_7}$

IS a partition.

NO elt is in TWO S_i
and
every elt is in some S_i